The Phantom of Differential Characteristics

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Motivation

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DISTINGUISHER $+$

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DISTINGUISHER + ATTACK

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For various application scenarios, we often assume the ability of an attacker to control the keys:

- Single-key model
- Open-key model
  - related-key attack
  - weak-key attack
  - known-key attack

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Differential cryptanalysis

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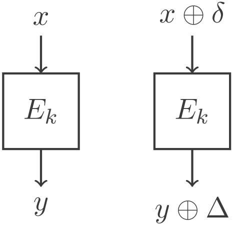
- One of the most extensively studied cryptanalytic techniques



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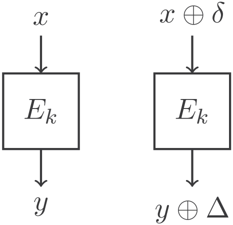

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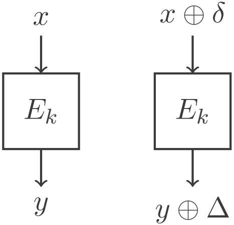

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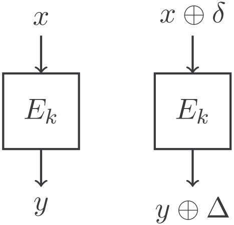
- Distinguish from random and key recovery



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Assumptions

- Markov cipher
- Independently random round keys
- Hypothesis of stochastic equivalence

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However, an attacker targets on one secret key.

- The probability of a differential distinguisher determines the attack complexity
- Differential or impossible differential?

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[Leu12] G. Leurant. Analysis of differential attacks in ARX constructions. ASIACRYPT 2012

[KNP+15] D. Khovratovich, I. Nikolić, J. Pieprzyk, P. Sokołowski, R. Steinfeld. Rotational cryptanalysis of ARX revisited. FSE 2015

[DR07] J. Daemen, V. Rijmen. Plateau characteristics. IET information security, 2007

[CLN+17] A. Canteaut, E. Lambooij, S. Neves, S. Rasoolzadeh, Y. Sasaki, M. Stevens. Refined Probability of Differential Characteristics Including Dependency Between Multiple Rounds. IACR ToSC 2017 (2)

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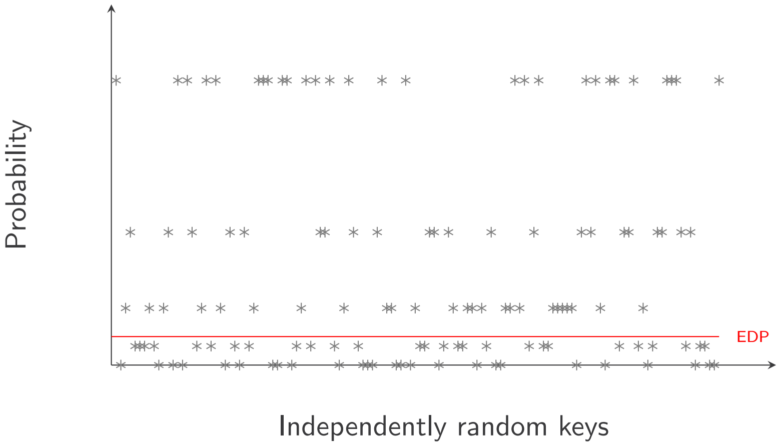

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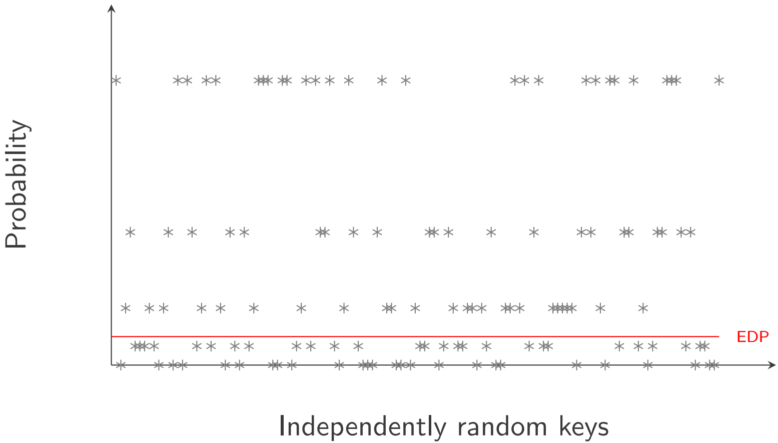

To what extent can we rely on the Assumptions?

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Enumerate characteristics under the Assumptions:

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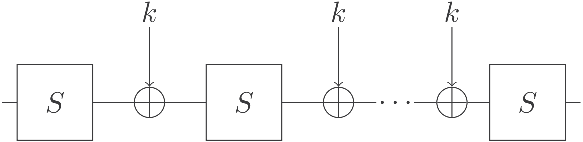

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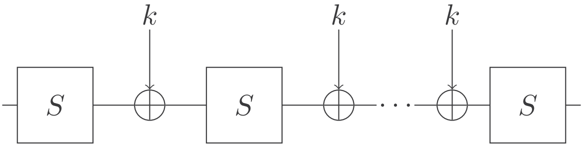

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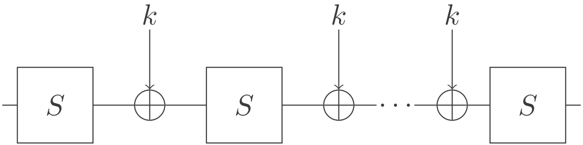

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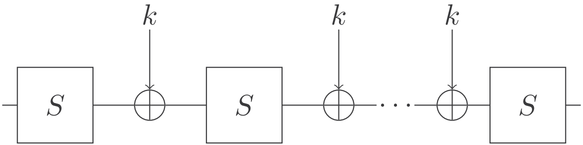

- For a fixed key, # characteristics $= 2^{15}$
- Under the Assumptions, # characteristics $=$ $2^8 \times 2^7 \times \cdots \times 2^7 = 2^{7r+8}$
- A characteristic generated under the Assumptions is "almost" impossible in reality.

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It is crucial to ask:

- EDP$\neq 0$ while DP $= 0$ for all keys?
- Differential characteristics enumeration?
- Characteristics-based attacks?
- Compute DP under any given key?
- Design better key schedules and/or constants?

Effective Keys and Singular Characteristics

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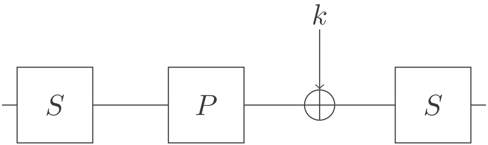
A key is effective for a characteristic if the characteristic is of nonzero probability under the key.

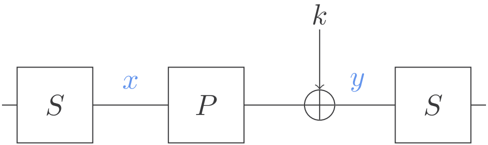
If no effective key exists, it is called a *singular characteristic*.

Effective Keys



- SPN cipher with keys XORed after the linear layer

Effective Keys



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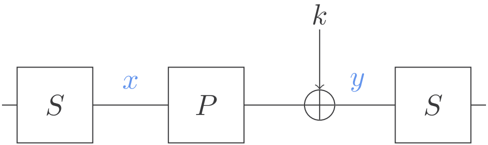
Effective Keys



- SPN cipher with keys XORed after the linear layer
- Right output and right input of the Sboxes
- Effective key candidates: $k = Px \oplus y$

Singular Characteristics

$$\alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4$$

Singular Characteristics

$$\underbrace{\alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1}_{K_1} \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4$$

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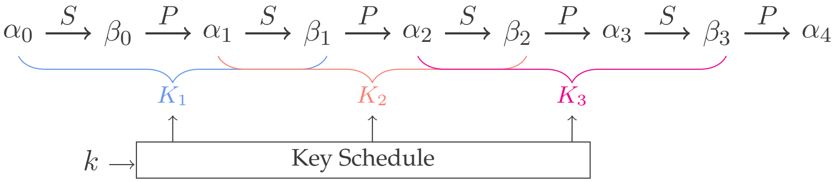

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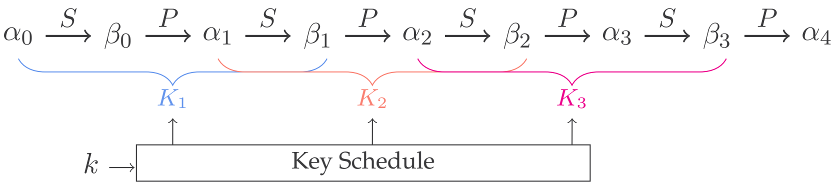

- When the difference propagation is legal, the effective key set of a 2-round characteristic is non-empty.

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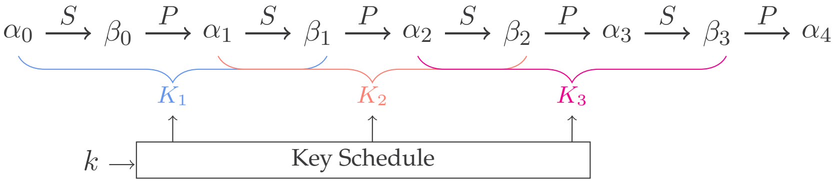
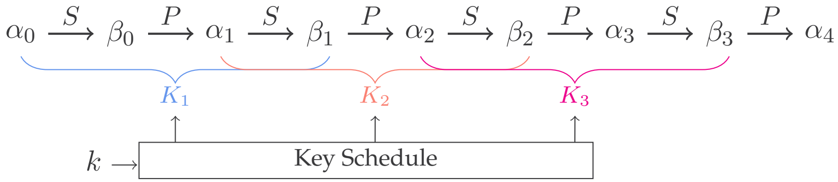


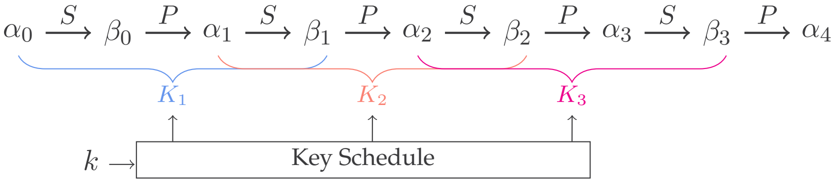
- When the difference propagation is legal, the effective key set of a 2-round characteristic is non-empty.
- Effective keys derived from two consecutive rounds may not be compatible with the key schedule.

Singular Characteristics



Procedure:

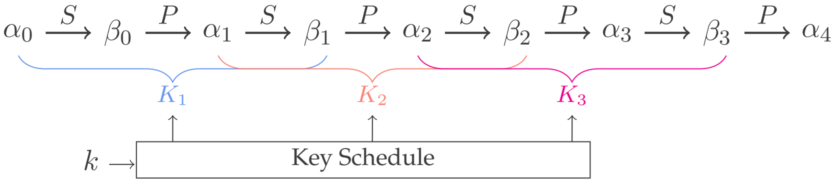
Singular Characteristics



Procedure:

1. Conditions on K_i to be effective

Singular Characteristics



Procedure:

1. Conditions on K_i to be effective
2. Conditions based on a specific key schedule

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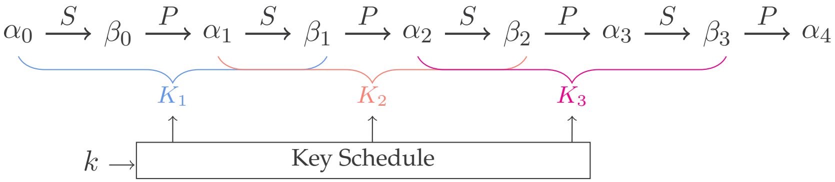
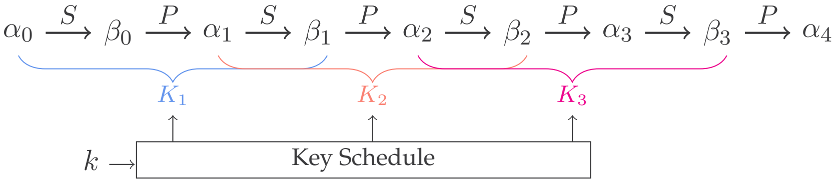


Procedure:

1. Conditions on K_i to be effective
2. Conditions based on a specific key schedule
3. Key schedule details

Singular Characteristics



Procedure:

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4. Linear equation systems

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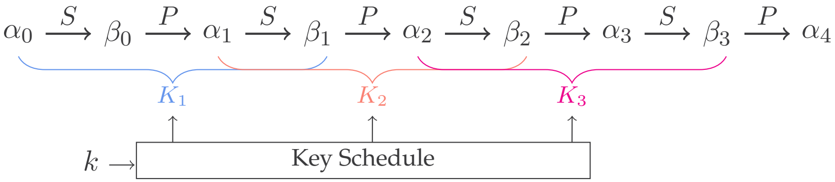

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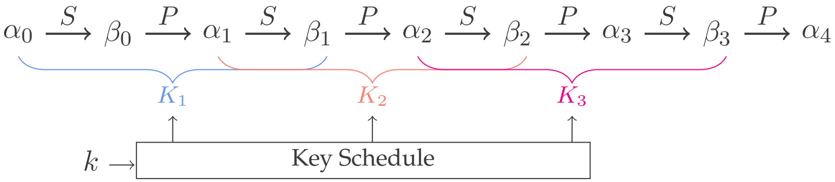

Procedure:

1. Conditions on K_i to be effective
2. Conditions based on a specific key schedule
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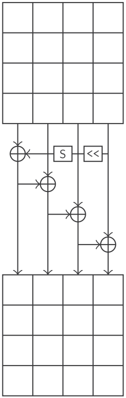
4. Linear equation systems
   - No solution found $\rightarrow$ singular
   - Key candidates found $\rightarrow$ Further filter by nonlinear constraints

Singular Characteristics in the AES

Find singular characteristics in AES-128:



Picture credit:
TikZ for Cryptographers

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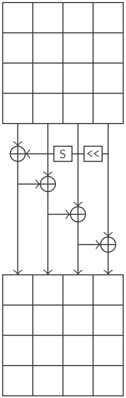
Find singular characteristics in AES-128:

- Subspaces of effective keys in every two consecutive rounds



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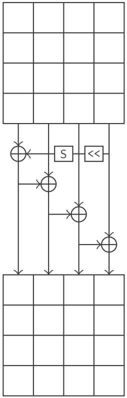

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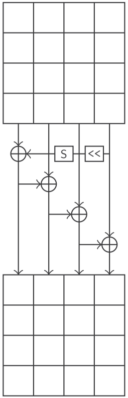

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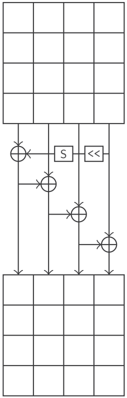
Singular Characteristics in the AES

Find singular characteristics in AES-128:

- Subspaces of effective keys in every two consecutive rounds
- Build equation systems with key schedule
- 3 out of 4 columns in AES-128 key schedule are linear relations
- Simplify and solve the equation system



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Singular Characteristics in the AES

Examples of 5-round singular characteristics can be found in the AES-128.

$$\begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \xrightarrow{P} \begin{pmatrix} 2&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 3&0&0&0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 3&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 2&0&0&0 \end{pmatrix}$$

$$\xrightarrow{P} \begin{pmatrix} 6&2&1&3\\ 3&2&3&2\\ 3&6&2&1\\ 5&4&1&1 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 24&27&39&9d\\ 45&36&36&27\\ 36&f1&2e&2d\\ 39&2d&1f&3a \end{pmatrix} \xrightarrow{P} \begin{pmatrix} 6&0&0&0\\ 0&5&0&0\\ 0&0&5&0\\ 0&0&0&36 \end{pmatrix}$$

$$\xrightarrow{S} \begin{pmatrix} e&0&0&0\\ 0&9&0&0\\ 0&0&d&0\\ 0&0&0&b \end{pmatrix} \xrightarrow{P} \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix}.$$

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$$\xrightarrow{P} \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \xrightarrow{S} \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \xrightarrow{P} \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix}$$

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MITM attack

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Density of singular characteristics:

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- Enumerate all characteristics given a 3-round differential

Singular Characteristics in the AES

Density of singular characteristics:

$$\begin{pmatrix} * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \end{pmatrix} \xrightarrow{P} \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \xrightarrow{S} \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \xrightarrow{P} \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{pmatrix} \xrightarrow{S} \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{pmatrix}$$

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- Enumerate all characteristics given a 3-round differential
- More than 98.47% of all the characteristics are singular
- For the remaining characteristics, we consider the nonlinear constraints from the key schedule and get their effective keys
  - some of them may also be singular
  - the number of effective keys is around 2^7 to 2^{10}

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- Extension to AES-like, Feistel-SP, Feistel

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$$\begin{pmatrix}8\,0\,4\,0\\0\,0\,0\,0\\4\,0\,8\,0\\0\,0\,0\,0\end{pmatrix} \xrightarrow{S} \begin{pmatrix}8\,0\,4\,0\\0\,0\,0\,0\\8\,0\,4\,0\\0\,0\,0\,0\end{pmatrix} \xrightarrow{M'} \begin{pmatrix}8\,0\,4\,0\\0\,0\,0\,0\\8\,0\,4\,0\\0\,0\,0\,0\end{pmatrix} \xrightarrow{SR} \begin{pmatrix}8\,0\,4\,0\\0\,0\,0\,0\\4\,0\,8\,0\\0\,0\,0\,0\end{pmatrix} \xrightarrow{S} \begin{pmatrix}8\,0\,5\,0\\0\,0\,0\,0\\8\,0\,5\,0\\0\,0\,0\,0\end{pmatrix}$$

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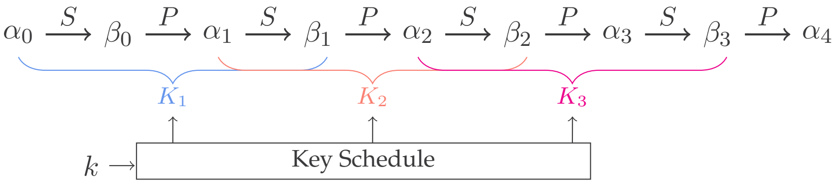
Singular Characteristics in Prince

$$
\begin{pmatrix} 8040 \\ 0000 \\ 4080 \\ 0000 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 8040 \\ 0000 \\ 8040 \\ 0000 \end{pmatrix} \xrightarrow{M'} \begin{pmatrix} 8040 \\ 0000 \\ 8040 \\ 0000 \end{pmatrix} \xrightarrow{SR} \begin{pmatrix} 8040 \\ 0000 \\ 4080 \\ 0000 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 8050 \\ 0000 \\ 8050 \\ 0000 \end{pmatrix}
$$

$$
\xrightarrow{M'} \begin{pmatrix} 8050 \\ 0000 \\ 8050 \\ 0000 \end{pmatrix} \xrightarrow{SR} \begin{pmatrix} 8050 \\ 0000 \\ 5080 \\ 0000 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 2050 \\ 0000 \\ 2050 \\ 0000 \end{pmatrix}
$$

A 3-round singular characteristic with EDP $= 2^{-35}$

Singular Cluster



Singular Cluster

$$\alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4$$

$$K_1 \qquad K_2 \qquad K_3$$

$$k \longrightarrow \boxed{\text{Key Schedule}}$$

$$K'_1 \qquad K'_2 \qquad K'_3$$

$$\alpha'_0 \xrightarrow{S} \beta'_0 \xrightarrow{P} \alpha'_1 \xrightarrow{S} \beta'_1 \xrightarrow{P} \alpha'_2 \xrightarrow{S} \beta'_2 \xrightarrow{P} \alpha'_3 \xrightarrow{S} \beta'_3 \xrightarrow{P} \alpha'_4$$

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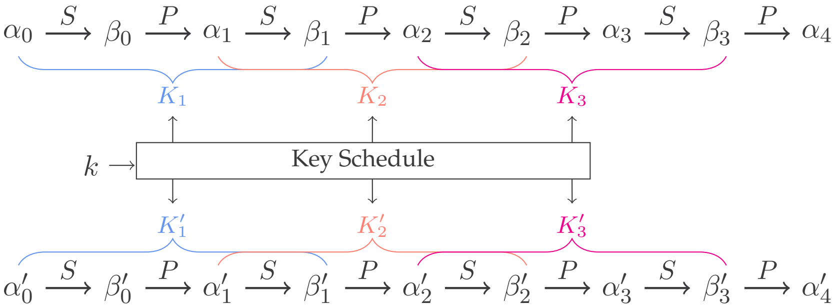

If no effective key in common $\rightarrow$ *singular cluster*.

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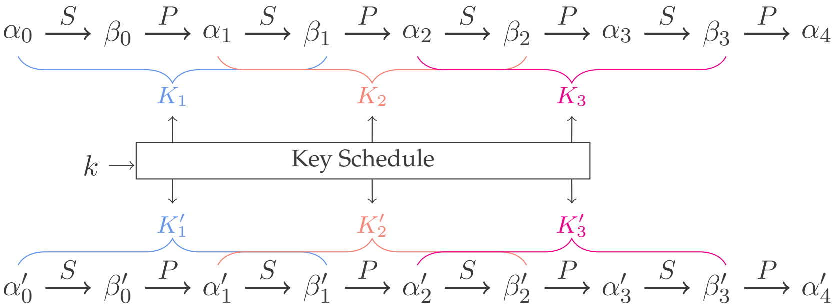

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Differentials/truncated differentials/multiple differentials

Further Applications

Observation: If a differential contains only singular characteristics, it is an impossible differential.

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- Improve distinguishers?

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Consider a 5-round differential $\mathcal{D}$ of the AES with active pattern 1-4-16-4-1. The effective keys of each characteristic can be precomputed.

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  - Exhaustive search space reduced?

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Thank you for your attention!